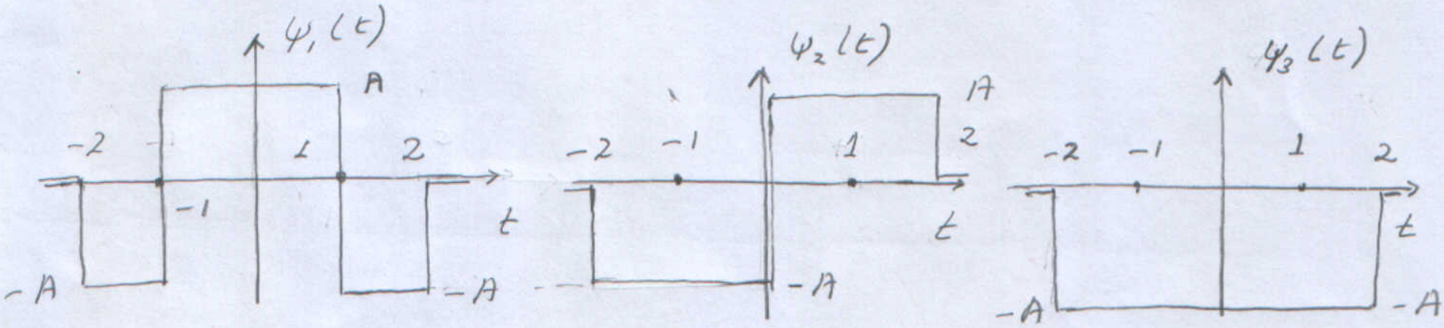
Çankaya University – ECE Department – ECE 376

Student Name :
Student Number :

Duration : 2 hours
Open book exam

Questions

1. (45 Points) Show that the waveforms, $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ shown below are orthogonal to each other. Find the value of A that will make $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ a set of orthonormalized basis functions. Express the following $s_m(t)$ (where m = 1 … M = 4) signals in terms of $\psi_i(t)$ (where i = 1 … 3). Plots all $s_m(t)$ signals on the signal space diagram. Write for $\mathbf{s_m}$ (vectors), find the energy in each $\mathbf{s_m}$. Suggest what type of modulation $s_m(t)$ waveforms will represent commenting on the efficiency of this representation. What is the minimum number of $s_m(t)$ signals to achieve a more efficient modulation ?



$s_1(t) = 1$ for $0 < t < 2$, $s_1(t) = 0$ elsewhere

$s_2(t) = 1$ for $-2 < t < 0$, $s_2(t) = 0$ elsewhere

$s_3(t) = 1$ for $-1 < t < +1$, $s_3(t) = 0$ elsewhere

$s_4(t) = 1$ for $-2 < t < -1$, $s_4(t) = 1$ for $1 < t < 2$, $s_4(t) = 0$ elsewhere

Solution: The orthogonality of $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ can be shown by taking the following

$$\int_{-2}^{2} \psi_i(t)\,\psi_k(t)\,dt = 0 \qquad \text{for } i = 1, 2, 3 \quad k = 1, 2, 3 \quad \text{and } i \neq k$$

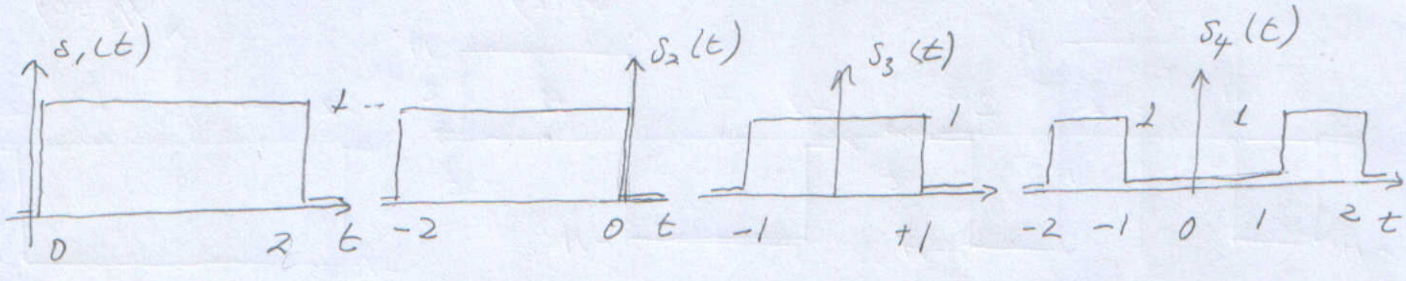
note that $T = 4$

In the case $i = k$, A can be found by setting the respective energy to unity, that is

$$\int_{-2}^{2} \psi_1^2(t)\,dt = \int_{-2}^{2} \psi^2(t)\,dt = \int_{-2}^{2} \psi_3^2(t)\,dt = 1$$

From these, $A = 0.5$

By plotting $s_1(t)$ --- $s_4(t)$ as time waveforms

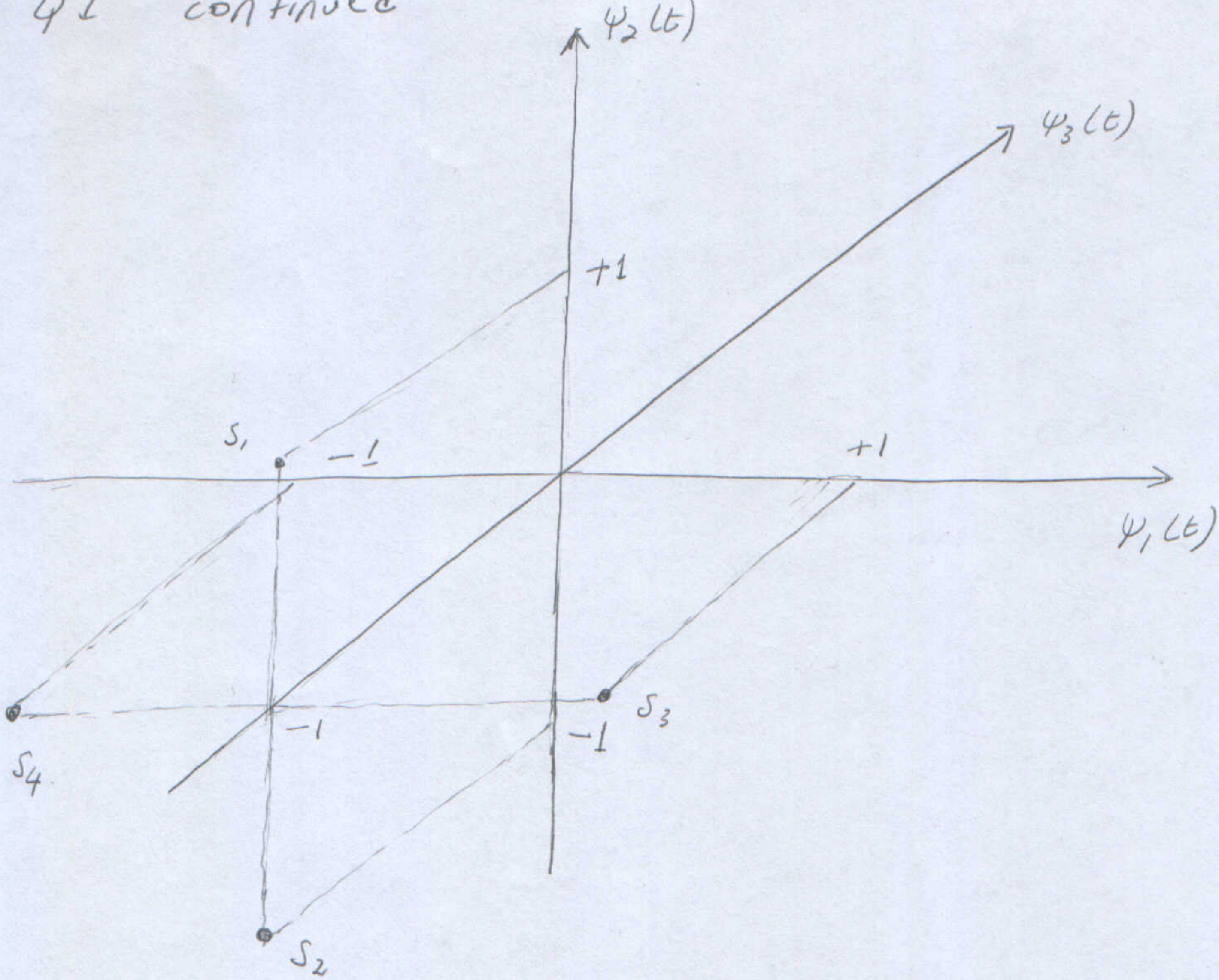


By eye inspection

Time Waveform	Vector	Energ
$s_1(t) = \psi_2(t) - \psi_3(t)$	$s_1 = [0 \;\; 1 \;\; -1]$	2
$s_2(t) = -\psi_2(t) - \psi_3(t)$	$s_2 = [0 \;\; -1 \;\; -1]$	2
$s_3(t) = \psi_1(t) - \psi_3(t)$	$s_3 = [1 \;\; 0 \;\; -1]$	2
$s_4(t) = -\psi_1(t) - \psi_3(t)$	$s_4 = [-1 \;\; 0 \;\; -1]$	2

Constellation diagram is shown on the next page

Q1 continued



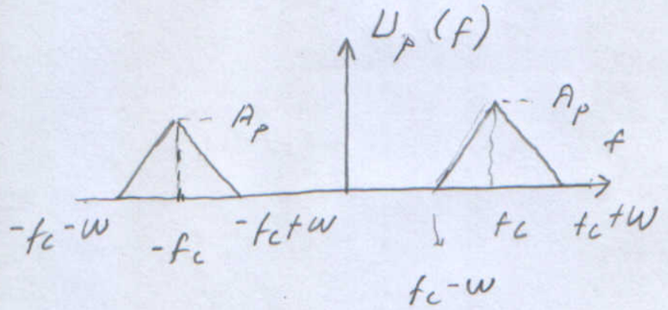
As seen, all vectors lie on the plane formed by $\psi_1(t) - \psi_2(t)$. The same can also be detected by examing $s_1(t)$ -- $s_4(t)$ or vectors S_1 -- S_4 since in these $\psi_3(t)$ is always present as $-\psi_3(t)$.

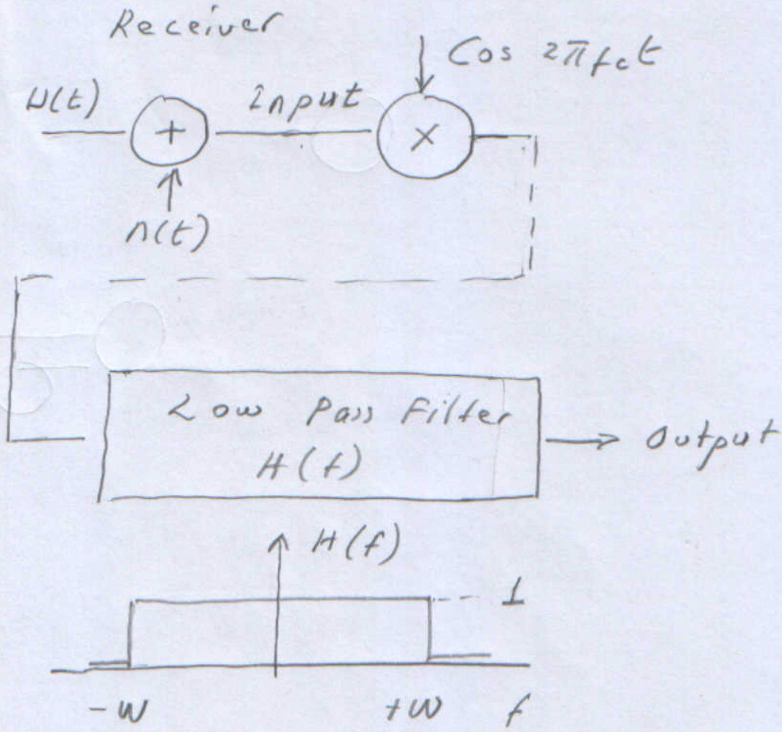
At first sight this looks like a three dimensional modulation (FSK or PPM), two dimesions are sufficient to represent $s_1(t)$ -- $s_4(t)$

In this case though basis functions and signal time waveforms would change.

To make a more efficient modulation while retaining the three dimensional structure, we would have to place four additional signals s_5 --- s_8 in the other vertices of the cube.

2. (25 Points) The DSB modulated signal $u(t)$ whose power spectral density $U_p(f)$ is as shown below arrives at the receiver mixed with AWGN (Additive White Gaussian Noise) with a spectral density of $N_0/2$. For demodulation, the setup on the right hand side is used. Find the input (before demodulation) and output (after demodulation) SNR for this receiver.





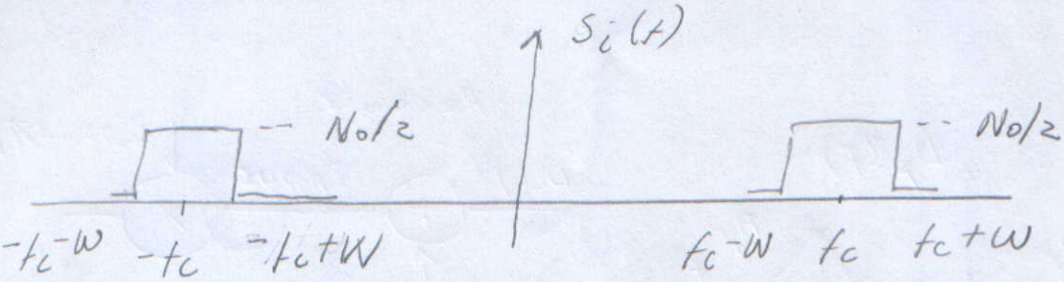
Solution: Power in the signal u(t) at input to receiver is

$$P_s = 4 \times \int_{-f_c-W}^{-f_c+W} \frac{A_p}{W}(f+f_c+W)\,df = 4\frac{A_p W}{2}$$

since powers in all triangles are identical, hence

$$P_s = 2A_pW$$

On the other band noise power at input to receiver



$$P_n = 2 \times \int_{-f_c-W}^{-f_c+W} \frac{N_0}{2}\, df = 2N_0W$$

$$SNR_{input} = \frac{2A_pW}{2N_0W} = \frac{A_p}{N_0}$$

After demodulation total power of the signal and the noise is reduced by $\frac{1}{2}$ due to multiplication by $\cos 2\pi f_c t$ $\left(P_{cos} = \frac{1}{2}\right)$

And the use of an $H(f) = 1$ for $|f| < W$ allows all half of the original signal and noise power spectrums to pass through Thus

$$SNR_{output} = \frac{A_p W}{N_0 W} = \frac{A_p}{N_0} = SNR\ input$$

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) FSK and PPM are the examples of multidimensional signals : True

FSK and PPM can be more than two dimensions

b) Binary PSK is equivalent to binary orthogonal FSK: False

Binary PSK equivalent to binary ASK in terms of error performance

c) Noise is *N* dimensional : False. Noise is infinite dimensional (before entering the correlator)

d) In QAM, there is amplitude variation of the signal within the symbol period : Partially true, but in PSK the same variation can be observed. A better description is, if different symbols are transmitted in consecutive symbol periods, then for QAM, the corresponding energies will be different

e) Signal constellation diagram shows the position of signal vectors as well as their amplitudes:

Partially true, constellation diagram showing the position of signal vectors is true but lengths of these signal vectors are only indications of energies, not necessaryly the amplitudes.